Deontic Sufficiency in Dyadic Deontic Logic

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Deontic Sufficiency

- Standard Deontic Logic (SDL): deontic reinterpretation of alethic modal logic
  - $\Box\varphi \rightsquigarrow O\varphi$
  - $\Diamond\varphi \rightsquigarrow P\varphi$
- Other modalities?
  - "Window" modality $\boxplus\varphi$: $(W, R, V), w \models \boxplus\varphi$ iff $\forall v(v \models \varphi \Rightarrow wRv)$
  - "The window $\boxplus$ may be pretty well interpreted as 'sufficiency', to the same extent at least to which 'necessity' is $\Box$ and 'possibility' is $\Diamond$." (Gargov et al., 1987)
- Deontic Sufficiency:
  - $\boxplus\varphi \rightsquigarrow S\varphi$: φ is a sufficient condition for satisfying all obligations/achieving ideality
  - $S\varphi$ as strong permission (Von Wright, 1971; Van Benthem, 1979; Van De Putte, 2017):

$$S(p \vee q) \leftrightarrow Sp \wedge Sq$$

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Dyadic Deontic Logic (Hansson, 1971)

- $O\varphi \rightsquigarrow O(\psi/\varphi)$
  - $O(\psi/\varphi)$: Given φ, it is obligatory that ψ
  - Contrary-to-duty paradoxes: Chisholm's
- $(W, R, V) \rightsquigarrow (W, \succeq, V)$
  - $w \succeq v$: w is at least as good as v
  - $O(\psi/\varphi)$ is true all the best φ-worlds are also ψ-worlds
- How to extend the deontic sufficiency modality to a dyadic setting?
  - $S(\psi/\varphi)$: Given φ, it suffices to do ψ to satisfy all obligations or achieve ideality
  - $S(\psi/\varphi)$ is true iff all the $\varphi \wedge \psi$-worlds are best φ-worlds

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Syntax and Semantics

- Language: $\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box\varphi \mid S(\psi/\varphi)$
  - $\Box\varphi$ (universal modality): φ is necessarily true
  - $S(\psi/\varphi)$: Given φ, it suffices to do ψ
- Model: $M = (W, \succeq, V)$
  - W: set of possible worlds (states) $\quad V : \text{PROP} \to \wp(W)$
  - $\succeq$: binary relation on W
- $M, w \models S(\psi/\varphi)$ iff $[\![\psi \wedge \varphi]\!]_M \subseteq opt_{\succeq}([\![\varphi]\!]_M)$

$$opt_{\succeq}([\![\varphi]\!]_M) = \{s \in [\![\varphi]\!]_M \mid s \succeq t \text{ for all } t \in [\![\varphi]\!]_M\}$$

$\Rightarrow$ In the context of φ, all ψ-worlds are best φ-worlds

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Example Validities

- Strengthening of the Antecedent: $\models S(\psi/\varphi) \to S(\psi/\varphi \wedge \chi)$
- Strengthening of the Consequent: $\models S(\psi/\varphi) \to S(\psi \wedge \chi/\varphi)$
- Conditional free choice: $\models S(\psi \vee \chi/\varphi) \leftrightarrow S(\psi/\varphi) \wedge S(\chi/\varphi)$
- Reasoning by cases:

$$S(\varphi/\varphi \vee \psi) \wedge S(\varphi/\varphi \vee \chi) \to S(\varphi/\varphi \vee \psi \vee \chi)$$

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Properties of $\succeq$

- A preference model is
  - *reflexive* if, for all $s \in W$, $s \succeq s$; $(\mathbb{R})$
  - *total* (or *connected*) if, for all $s, t \in W$, $s \succeq t$ or $t \succeq s$; $(\mathbb{C})$
  - *transitive* if, for all $s, t, w \in W$, $s \succeq t$ and $t \succeq w$ implies $s \succeq w$; $(\mathbb{T})$
  - *limited* if, for all formulas φ, $[\![\varphi]\!]_M \neq \varnothing$ implies $opt_{\succeq}([\![\varphi]\!]_M) \neq \varnothing$. $(\mathbb{L})$
- $\mathbb{RTL}$ denotes the class of all reflexive, transitive, and limited models, and likewise for other subsets of $\{\mathbb{R}, \mathbb{C}, \mathbb{T}, \mathbb{L}\}$
- $\mathrm{L}_{\mathbb{RTL}}$: the set of all validities on the model class $\mathbb{RTL}$; similarly for other subsets of $\{\mathbb{R}, \mathbb{C}, \mathbb{T}, \mathbb{L}\}$

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The logics of deontic sufficiency on different modal classes

We comprehensively studied the logics generated by the 16 (possibly equivalent) classes of models:

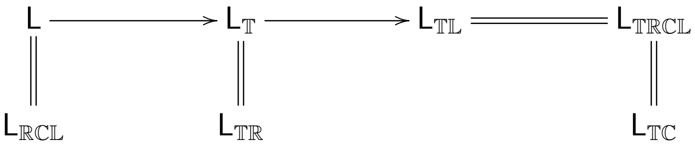

Figure: The logics generated by different model classes, where $\mathsf{L}_X = \mathsf{L}_Y$ means that the two logics are the same and $\mathsf{L}_X \longrightarrow \mathsf{L}_Y$ means that L_X is a proper subset of L_Y.

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Axiomatizations

PL	All instances of propositional tautologies
$\Box$-K	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$
$\Box$-T	$\Box\varphi \to \varphi$
$\Box$-5	$\neg\Box\varphi \to \Box\neg\Box\varphi$
A1	$S(\psi/\varphi) \to \Box S(\psi/\varphi)$
A2	$S(\psi/\varphi) \wedge S(\chi/\varphi) \to S(\psi \vee \chi/\varphi)$
A3	$S(\varphi/\varphi \vee \psi) \to (S(\varphi/\varphi \vee \chi) \to S(\varphi/\varphi \vee \psi \vee \chi))$
A4	$\Box(\psi \to \chi) \to (S(\chi/\varphi) \to S(\psi/\varphi))$
A5	$\Box(\psi \to \varphi) \to (S(\chi/\varphi) \to S(\chi/\psi))$
A6	$\Box\neg(\psi \wedge \varphi) \to S(\psi/\varphi)$
MP	From φ and $\varphi \to \psi$, infer ψ
Nec	From φ, infer $\Box\varphi$

The axiomatization $\mathbf{DLDS}_0$

Axiomatizations

- $\mathbf{DLDS}_1 = \mathbf{DLDS}_0 + (\text{Tran})$:

$$S(\varphi/\varphi \vee \psi) \wedge S(\psi/\psi \vee \chi) \wedge \Diamond\psi \rightarrow S(\varphi/\varphi \vee \chi) \qquad (\text{Tran})$$

- $\mathbf{DLDS}_2 = \mathbf{DLDS}_1 + (\text{Lim})$:

$$S(\varphi/\psi) \wedge S(\chi/\theta) \rightarrow (S(\varphi \wedge \psi/\psi \vee \theta) \vee S(\chi \wedge \theta/\psi \vee \theta)) \qquad (\text{Lim})$$

- Summary of completeness results:

$$\begin{array}{ccccccc}
\mathsf{L}(= \mathbf{DLDS}_0) & \longrightarrow & \mathsf{L}_{\mathbb{T}}(= \mathbf{DLDS}_1) & \longrightarrow & \mathsf{L}_{\mathbb{TL}}(= \mathbf{DLDS}_2) & = & \mathsf{L}_{\mathbb{TRCL}} \\
\| & & \| & & & & \| \\
\mathsf{L}_{\mathbb{RCL}} & & \mathsf{L}_{\mathbb{TR}} & & & & \mathsf{L}_{\mathbb{TC}}
\end{array}$$

- $\mathbf{DLDS}_0$ – $\mathbf{DLDS}_2$ are all decidable.

Completeness problem in DDL

- The completeness problem of (some) dyadic deontic logics has been settled only recently (Parent, 2015). In certain cases, the construction of canonical model is rather involved.
- "Unfortunately, the connection between axioms on the one hand and constraints on the other is not as straightforward as the correspondence results for standard modal logic. ...In general, the correspondence theory of these languages remains an open area of research with only a few results available so far." (Grossi et al., 2022)
- Our paper proposes a novel **modular** method for proving **weak** completeness in dyadic deontic logic

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Summary and Future Work

Summary:

- introduce and study the dyadic logics of deontic sufficiency (conditional version of strong permission)
- comprehensive study of the logics on different modal classes

Future work:

- dyadic logic of both deontic necessity and sufficiency: $O(\psi/\varphi) + S(\psi/\varphi)$
- maximality version of $S(\psi/\varphi)$: $[\![\varphi \wedge \psi]\!]_M \subseteq max_{\succeq}([\![\varphi]\!]_M)$
- Input/Output Logic for conditional strong permission

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