Reasoning about Epistemic Rights

Xu Li
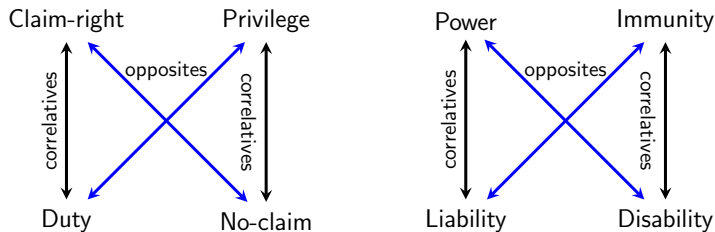
University of Luxembourg

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Introduction

- Epistemic rights are rights concerning epistemic goods: knowledge, truth, belief, justification, ignorance, ...
  - ⇒ **The right to know**, the right to truth, the right to believe, the right to explanation, ...
- Epistemic rights (the right to know) are rights:

- The right to know: the claim/privilege/power/immunity to know
- Logics for reasoning about epistemic rights (the right to know) in DELIGHT

The claim-right to know and obligatory announcements

- x has the claim-right to know φ against y
  - $\Rightarrow$ y has the duty toward x (to see to it) that x knows φ
  - $\Rightarrow$ y has the duty to inform x about φ (obligatory announcements)
- Logics of Permitted and Obligatory Announcements (LPOA) [1]
- Main challenge of formalizing perm. and obl. announcements:

  interplay between the sender's (y) perm. and obl. announcements and the receiver's (x) knowledge

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[1]Xu Li, Guillaume Aucher, Dov Gabbay, and Réka Markovich. "From Knowledge to Action: Logics of Permitted and Obligatory Announcements". In: *Journal of Artificial Intelligence Research* 82.2025 (2025), pp. 1629–1672

Example: Research Funding Fraud

- In 2020, a scientist was found to be involved in a research funding fraud by the European Anti-Fraud Office (OLAF)
- OLAF published a press release on its website which contains the scientist's nationality and gender, his or her father's occupation, the amount of the grant, . . .
- The scientist sued OLAF for unlawful disclosure of personal data
- Whether the information in the press release constitutes personal data?

> 'personal data' means any information relating to an identified or identifiable natural person ('data subject') . . .   (Article 4(1) of GDPR)

- Reasoning about the receiver's knowledge!

Case C-479/22 P from the Court of Justice of the European Union.

Language of LPOA: weakly and strongly perm. announcements

- $\varphi ::= p \mid \neg\varphi \mid (\varphi \to \varphi) \mid K\varphi \mid \mathbb{O}^-\varphi \mid \mathbb{P}^-\varphi \mid \mathbb{P}^+\varphi \mid [\varphi]\varphi$
  $\Rightarrow \mathbb{O}^+\varphi := \mathbb{O}^-\varphi \wedge \mathbb{P}^+\varphi$

Operator	Interpretation	Reading
$\mathbb{O}^-\varphi$	Announcing φ is **necessary** for complying with the security policies	It is **obligatory** for the sender to (truthfully) announce φ
$\mathbb{P}^+\varphi$	Announcing φ is **sufficient** for . . .	It is **strongly permitted** for . . .
$\mathbb{P}^-\varphi$	Announcing φ is **not forbidden**	It is **weakly permitted** for. . .
$\mathbb{O}^+\varphi$	Announcing φ is both **sufficient and necessary** for . . .	φ is the least informative (strongly) permitted announcement

Semantics of LPOA: ideal epistemic states

- A model is a tuple $M = (W, N, V)$ with $N : W \to \wp(\wp(W))$
  - $Y \in N(w)$: Y an ideal epistemic state (for the receiver) relative to w
  - for all w and $Y \in N(w)$, $w \in Y$
- Formulas are evaluated w.r.t pairs (w, X) with $w \in X \subseteq W$

$$M, w, X \models K\varphi \quad \text{iff} \quad \text{for all } v \in X, \ M, v, X \models \varphi$$

$$M, w, X \models \mathbb{P}^+\varphi \quad \text{iff} \quad [\![\varphi]\!]_{M,X} \in N(w)$$

$$M, w, X \models \mathbb{P}^-\varphi \quad \text{iff} \quad \text{there is } Y \in N(w) \text{ such that } Y \subseteq [\![\varphi]\!]_{M,X}$$

$$M, w, X \models \mathbb{O}^-\varphi \quad \text{iff} \quad M, w, X \models \varphi \text{ and for all } Y \in N(w),$$
$$Y \subseteq X \text{ implies } Y \subseteq [\![\varphi]\!]_{M,X}$$

$$M, w, X \models [\varphi]\psi \quad \text{iff} \quad M, w, X \models \varphi \text{ implies } M, w, [\![\varphi]\!]_{M,X} \models \psi$$

Axiomatization of LPOA

- No reduction axiom exists for $[\varphi]\mathbb{O}^-\psi$

Axioms:

(PL)	All propositional tautologies	(!Atom)	$[\varphi]p \leftrightarrow (\varphi \to p)$
(S5)	S5 axioms for K	(!Neg)	$[\varphi]\neg\psi \leftrightarrow (\varphi \to \neg[\varphi]\psi)$
(A5)	$K(\varphi \leftrightarrow \psi) \to (\mathbb{P}^+\varphi \leftrightarrow \mathbb{P}^+\psi)$	(!Imp)	$[\varphi](\psi \to \chi) \leftrightarrow ([\varphi]\psi \to [\varphi]\chi)$
(A8)	$\mathbb{O}^-\varphi \to (\mathbb{P}^+\psi \to K(\psi \to \varphi))$	(!K)	$[\varphi]K\psi \leftrightarrow (\varphi \to K[\varphi]\psi)$
(A9)	$\neg\mathbb{P}^-\varphi \to (\mathbb{P}^+\psi \to \neg K(\psi \to \varphi))$	(!$\mathbb{P}^+$)	$[\varphi]\mathbb{P}^+\psi \leftrightarrow (\varphi \to \mathbb{P}^+\langle\varphi\rangle\psi)$
(A10)	$\mathbb{P}^+\varphi \to \varphi$	(!$\mathbb{P}^-$)	$[\varphi]\mathbb{P}^-\psi \leftrightarrow (\varphi \to \mathbb{P}^-\langle\varphi\rangle\psi)$
(A11)	$\mathbb{O}^-\varphi \to \varphi$	(!Comp)	$[\varphi][\psi]\chi \leftrightarrow [\langle\varphi\rangle\psi]\chi$

Rules:

(MP)	from φ and $\varphi \to \psi$, infer ψ
(Nec$_K$)	from φ, infer $K\varphi$
(Nec$_{[]}$)	from φ, infer $[\psi]\varphi$
(R$_{\mathbb{O}-}$)	from $\xi(\mathbb{P}^+p \to K(p \to \varphi))$, infer $\xi(\varphi \to \mathbb{O}^-\varphi)$, where $p \notin \mathrm{PROP}(\xi(\varphi))$
(R$_{\mathbb{P}-}$)	from $\xi(\mathbb{P}^+p \to \neg K(p \to \varphi))$, infer $\xi(\neg\mathbb{P}^-\varphi)$, where $p \notin \mathrm{PROP}(\xi(\varphi))$
(R$_{\mathbb{P}+}$)	from $\xi(\mathbb{P}^+p \to \neg K(p \leftrightarrow \varphi))$, infer $\xi(\neg\mathbb{P}^+\varphi)$, where $p \notin \mathrm{PROP}(\xi(\varphi))$

- $\vdash \mathbb{O}^-\varphi \to (\mathbb{P}^+\psi \to K(\psi \to \varphi))$

  $\Rightarrow$ An obligatory announcement φ is less informative than any strongly permitted announcement ψ

- $$\frac{\vdash \mathbb{P}^+p \to K(p \to \varphi), \ p \text{ does not apprear in } \varphi}{\vdash \varphi \to \mathbb{O}^-\varphi}$$

  $\Rightarrow$ If it can be proved that φ is less informative than an arbitrary strongly permitted announcement p, then it can be proved that φ is an obligatory announcement (provided that φ is true).

Non-S5 epistemic logic and future work

- What if the underlying epistemic logic is non-S5?
- Different choices of the underlying epistemic logic give rise to different logics of perm. and obl. announcements!

Future work:

- Strong completeness, compactness, strong completeness, decidability?
- The multi-agent case
- Probabilistic announcements:

> **Example:**[2] We want to collect the answers to some embarrassing question (e.g., "Have you ever cheated on your partner?") for some statistic purpose. To persuade people to answer truly, we allow them to report the true answer with probability 3/4, and the opposite answer with probability 1/4. In this way, the privacy of the user will be protected ... At the same time, if the population is large enough, the collector will be able to obtain a good statistical approximation of the real percentage of cheaters.

[2]Valentina Castiglioni, Konstantinos Chatzikokolakis, and Catuscia Palamidessi. "A logical characterization of differential privacy". In: *Science of Computer Programming* 188 (2020), p. 102388

Power-right to know and right of access

- x has the power-right to know φ against y:
  - $\Rightarrow$ y has the duty to inform x about φ if x requests it
- The right of access (Article 15(1) of GDPR):

> The data subject shall have the right to obtain from the controller confirmation as to whether personal data concerning him or her are being processed and, where that is the case, access to the personal data and the following information:
>
> . . .
>
> h. the existence of automated decision-making, including profiling, . . . and, at least in those cases, meaningful information about the logic involved . . .

Example: Mobile Phone Contract

- An Austrian individual was denied a mobile phone contract following a fully automated credit assessment.
- The automated assessment concluded that the individual lacked the financial capacity to pay the €10 monthly fee.
- To understand the rationale behind the decision, the individual requested meaningful information about the logic involved in the automated processing under Article 15(1)(h) of the EU GDPR.
- However, the mobile phone company refused, citing the protection of trade secrets under Directive 2016/943 in respect of their AI algorithms.
- The court found that the company infringed Article 15(1)(h) of the GDPR by failing to provide with meaningful information about the logic involved in the automated decision-making.

Case C-203/22 from the Court of Justice of the European Union.

Language of the logic of power-right to know (LRK)

- $\varphi ::= p \mid \neg\varphi \mid (\varphi \to \varphi) \mid U\varphi \mid K_r\varphi \mid R_r\varphi \mid \mathbb{O}_s\varphi \mid [r\!:\!\varphi?]\varphi \mid [s\!:\!\varphi!]\varphi$
- Two new operators:
    - $R_r\varphi$: The receiver has the power-right to know whether φ
    - $[r\!:\!\varphi?]\psi$: After the receiver asked the question φ?, it holds that ψ
- Other operators: $K_r\varphi$, $\mathbb{O}_s\varphi$, and $[s\!:\!\varphi!]\psi$

Semantics

- A model is a tuple $M = (W, \sim, \approx, N, V)$ where:
  - $\sim$: equivalence relation on W, representing the receiver's knowledge
  - $\approx$: a partition of W, encoding the set of questions to which the receiver has the power-right to know the answers
  - $N : W \to \wp(\wp(W))$, same as before
- $M, w \models R_r \varphi$ iff   for all $U \in \approx$, $U \subseteq [\![\varphi]\!]_M$ or $U \subseteq [\![\neg\varphi]\!]_M$
- $M, w \models [r\!:\!\varphi?]\psi$ iff $M_{\varphi?}, w \models \psi$

$$M_{\varphi?} = \begin{cases} M & \text{if } M, w \not\models R_r \varphi \\ (W, \sim, \approx, N_{\varphi?}, V) & \text{otherwise} \end{cases}$$

where $N_{\varphi?}(x) = \{U \in N(x) \mid U \subseteq [\![\varphi]\!]_M \text{ or } U \subseteq [\![\neg\varphi]\!]_M\}$ for all x

(all the epistemic states not answering the question φ? become no longer ideal)

Axiomatization and example (in)validities

- The logic has been completely axiomatized

- Example (in)validities:

  - $R_r\varphi \to R_r\neg\varphi$
    $R_r\varphi \land R_r\psi \to R_r(\varphi \land \psi)$

  - $\models R_r\varphi \to (\varphi \to [r:\varphi?]\mathbb{O}_s\varphi)$ and
    $\models R_r\varphi \to (\neg\varphi \to [r:\varphi?]\mathbb{O}_s\neg\varphi)$, where φ a propositional formula

  - $\not\models R_r\varphi \to (\varphi \to [r:\varphi?]\mathbb{O}_s\varphi)$ if, e.g., $\varphi = p \land \neg\mathbb{O}_s p$

  - $\models \neg R_r\varphi \to ([r:\varphi?]\mathbb{O}_s\psi \leftrightarrow \mathbb{O}_s\psi)$

A problem with the semantics of $[r : \varphi?]$

- $\models \neg R_r \varphi \rightarrow ([r : \varphi?]\mathbb{O}_s \psi \leftrightarrow \mathbb{O}_s \psi)$
- Counter-example:
    - Suppose r has the right to know whether p holds, but is forbidden to know q. Moreover, both p and q are true.
    - Thus, r does not have the right to know whether the conjunction $p \wedge q$ holds. $(\neg R_r(p \wedge q))$
    - The sender has no obligation to announce p now. $(\neg \mathbb{O}_s p)$
    - What if the receiver asks the question $p \wedge q$?
    - Then the sender has the obligation to announce p $([r : p \wedge q?]\mathbb{O}_s p)$
    - This cannot be modelled in LRK: $\models \neg R_r(p \wedge q) \rightarrow ([r : p \wedge q?]\mathbb{O}_s p \leftrightarrow \mathbb{O}_s p)$
- A fine-grained semantic analysis of the relationship between the power to know, questions, and obligatory announcements

The privilege to know and epistemic obligation

- x has the privilege to know φ (against y)
  - $\Rightarrow$ x has no duty (toward y) not to know φ
  - $\Rightarrow$ x is permitted to know φ (against y)
- $\neg O \neg K_x \varphi$: combine the languages of SDL and EL
- Åqvist's paradox of epistemic obligation: $OK_i \varphi \rightarrow O\varphi$
- Epistemic obligation and actuality:

  epistemic obligation, unlike the obligation studied in SDL, is closely tied to what is actually true

  - $\Rightarrow$ $Op \rightarrow p$ $(\times)$
  - $\Rightarrow$ $OK_i p \rightarrow p$ $(\checkmark)$
  - $\Rightarrow$ $O(K_i p \vee K_i \neg p) \rightarrow (p \rightarrow OK_i p)$ $(\checkmark)$
  - $O(K_i p \vee K_i \neg p) \rightarrow (\neg p \rightarrow OK_i \neg p)$ $(\checkmark)$

Solution:

- Analyzing knowledge in terms of necessary conditions, e.g., justified true belief
- In epistemic obligation $OK_i\varphi$, the truth condition of knowledge does not fall in the scope of the obligation modality O[3]
- How to deal with the scope issue with Kripke semantics?
- Two-dimensional semantics! E.g., the "now" operator in temporal logic[4]

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[3]Risto Hilpinen. "On the Sick Father, the Repentant Sinner, and Other Problems in Medieval Deontic Logic". In: *Theoria* 85.6 (2019), pp. 420–434
[4]Hans Kamp. "Formal properties of 'now'". In: *Theoria* 37.3 (1971), pp. 227–273

Summary and future work

Summary:

- Different meanings of "right to know" (theory of normative positions)
- Logic of perm. and obl. announcements (LPOA): claim-right to know
- Logic of power-right to know (LRK): power-right to know/the right of access
- Challenges of formalizing epistemic obligation (the privilege to know)

Summary and future work

Future work:

- Combine LPOA with LRK (fine-grained semantics for $[r:\varphi?]$)
- Decidability?
- The multi-agent and probabilistic extensions
- Epistemic rights beyond the right to know:
  the right not to know, the right to be forgotten, and the right to explanation, . . .

Thanks for your attention!
Any questions are welcome.

Reference

[1] Valentina Castiglioni, Konstantinos Chatzikokolakis, and Catuscia Palamidessi. "A logical characterization of differential privacy". In: *Science of Computer Programming* 188 (2020), p. 102388.

[2] Risto Hilpinen. "On the Sick Father, the Repentant Sinner, and Other Problems in Medieval Deontic Logic". In: *Theoria* 85.6 (2019), pp. 420–434.

[3] Hans Kamp. "Formal properties of 'now'". In: *Theoria* 37.3 (1971), pp. 227–273.

[4] Xu Li and Réka Markovich. "A Dynamic Logic of the Right to Know". In: *Journal of Applied Logics* 12.2 (Feb. 2025), pp. 221–250.

[5] Xu Li et al. "From Knowledge to Action: Logics of Permitted and Obligatory Announcements". In: *Journal of Artificial Intelligence Research* 82.2025 (2025), pp. 1629–1672.